Turbulent Equipartition and Homogenization of Plasma Angular Momentum

Ö. D. Gürcan* and P. H. Diamond
Center for Astrophysics and Space Sciences, University of California, San Diego, 9500 Gilman Drive, La Jolla, California 92093-0424, USA

T. S. Hahm
Princeton Plasma Physics Laboratory, Princeton University, Princeton, New Jersey 08543-0451, USA
(Received 13 September 2007; published 3 April 2008)

A physical model of turbulent equipartition (TEP) of plasma angular momentum is developed. We show that using a simple, model insensitive ansatz of conservation of *total* angular momentum, a TEP pinch of angular momentum can be obtained. We note that this term corresponds to a part of the pinch velocity previously calculated using quasilinear gyrokinetic theory. We observe that the nondiffusive TEP flux is inward, and therefore may explain the peakedness of the rotation profiles observed in certain experiments. Similar expressions for linear toroidal momentum and flow are computed and it is noted that there is an additional effect due the radial profile of moment of inertia density.

DOI: 10.1103/PhysRevLett.100.135001 PACS numbers: 52.35.Ra

The question of angular momentum transport in magnetic fusion devices is decades old. The theory traditionally focuses on description of angular momentum evolution in the presence of external torques [1]. Early theoretical predictions that $\chi_\phi \sim \chi_i$ were based on the structure and properties of long wavelength drift-wave turbulence [2]. Subsequent observations [3] indicate that anomalous angular momentum transport is important for tokamaks. Recent observations of intrinsic plasma rotation, its current scaling [4–7] or correlation with pressure gradients [8] observed in the high confinement regime (H mode) and the density bifurcations [9] observed in the low confinement regime (L mode), suggest that this simple picture of angular momentum transport is incomplete. In particular a nondiffusive component of the angular momentum flux is required. Recently, several theories explaining various aspects of these observations have been proposed. One particular idea, using $E \times B$ shear as the source of imbalance in the wave population density propagation direction, predicts a residual stress component of the momentum flux [10] mostly localized to the pedestal region. This may help explain intrinsic rotation via the interaction with sharp edge pressure and density gradients. However, in some cases, the rotation profiles are peaked on the plasma axis. Thus, a "pinch" (an inward radial flow of momentum, especially active in the core region) is also needed in order to explain these observations. A generic reformulation of this theory where the source of symmetry breaking is general (several possibilities are considered) indeed results in such a pinch term [11]. This effect is linked to wave-particle momentum exchange and therefore is independent of the details of geometry. Note that, whatever the reason may be, some symmetry breaking (in parallel wave number $k_\|$, interpreted broadly) is needed in order to have a nonvanishing off-diagonal component of the toroidal Reynolds stress.

Nonlinear gyrokinetic theory that respects the conservation properties of the Vlasov equation [12,13] yields such a pinch term when the parallel velocity moment is computed [14]. This has been interpreted as symmetry breaking in $\mathbf{B}^*$ due to ballooning structure of fluctuations [14] or as a manifestation of Coriolis drift in rotating frame [15]. Here we use turbulent equipartition theory (TEP) [16–18] and turbulent homogenization of angular momentum to derive a simple model which includes a similar radial convection term. Here homogenization refers to the process of flattening of the gradient of a scalar quantity that is advected by an incompressible flow (a locally conserved field) and diffused by molecular or turbulent diffusion [19]. Local differential rotation, such as that caused by zonal flows [20], ubiquitous in fusion plasmas, tends to speed up this process of homogenization [21]. This approach allows us to clarify the physical basis of this geometry dependent term. In a simple, model insensitive ansatz, we will show that this term corresponds to a part of the previous gyrokinetic result but follows from a simple physical explanation. We will show that the same form for the TEP pinch can be obtained from the simple assertion that $L_\phi/\lambda_1(B)$ is locally conserved. Here L_ϕ is the angular momentum density and λ_1 is a function of the magnetic field, which depends on geometry and/or plasma dynamics.

There are three aspects of the mechanism that are discussed in this Letter. First, there is the familiar TEP pinch. Just like particle number, angular momentum is also conserved by the underlying physical processes. Hence, being a "density," L_ϕ, the angular momentum density, diffuses in the flux surface variable ψ (just like the particle density). This suggests that it is roughly L_ϕ/λ_1 that is "locally conserved" by the $E \times B$ flow due to the fact that it is $u \equiv v_{E\times B}\lambda_1$, which is divergence free, and not the $E \times B$ flow itself. This results in a pinch of angular momentum, equivalent to the TEP particle pinch (the ratio of two is $V_r^{(n)}/V_r^{(L)} \sim D_n/\chi_\phi$).

The second aspect of the mechanism is linked to the relation between angular momentum and linear toroidal momentum (i.e., P_ϕ). In general, $L_\phi = (\langle R^2\rangle/\langle R\rangle)P_\phi \equiv \lambda_2^{-1}P_\phi$, and the proportionality (i.e., $\lambda_2^{-1} = \langle R^2\rangle/\langle R\rangle$) is related to the effective moment of inertia and is an increasing function of r (i.e., minor radius). For a simple torus with concentric circular flux surfaces $mn(\psi)\langle R^2\rangle \rightarrow I'(\psi)/V'(\psi) \rightarrow mn(r)(R_0^2 + 3r^2/2)$, which suggests that the core is less inert (i.e., lighter) than the edge, so if the angular momentum is homogenized by turbulence (and we will show that it is in fact L_ϕ/λ_1 that is homogenized), the rotation near the core will be faster than at the edge (see Fig. 1). Note that here $I(\psi)$ is the moment of inertia density. Because of this "radial profile" of the moment of inertia density in toroidal geometry, linear momentum evolution does not have the form of divergence of a flux. One can put it in such a form (approximately) after integrating by parts, where it can be observed that $P_\phi/(\lambda_1\lambda_2^2)$ is the locally conserved field.

The final aspect of the mechanism involves the transformation $P_\phi \rightarrow nv_\phi$ and the fact that it is v_ϕ that is actually measured. Decoupling the density and the flow evolutions leads to a recoil effect of Γ_n on v_ϕ evolution. It should be noted that the geometric part of the pinch effect (i.e., the part due to the divergence of $E \times B$ flow) is mainly a result of L_ϕ being a density of angular momentum. However v_ϕ is not a density. Thus, the main pinch effect on v_ϕ is due to the moment of inertia profile.

The TEP theory can be briefly reviewed using the equation of continuity:

$$\partial_t n + \nabla\cdot(n\mathbf{v}) = 0 \tag{1}$$

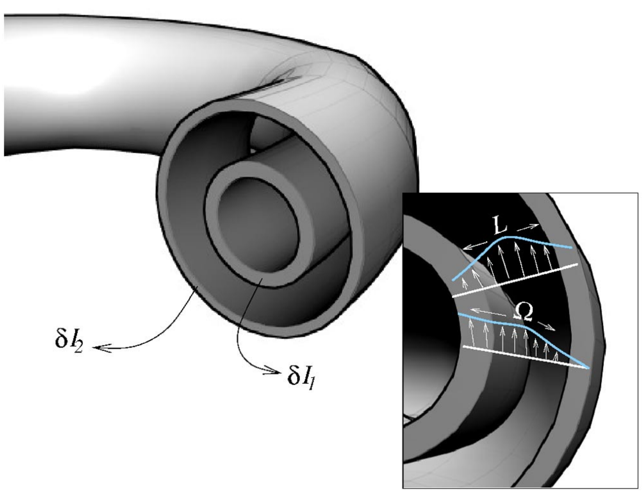

FIG. 1 (color online). Two toroidal shells of moment of inertia densities δI_1 and δI_2. Since $\delta I_1 < \delta I_2$, the core has less moment of inertia density than the edge; as a result, when the momentum is homogenized, the core rotates faster.

and following Naulin *et al.* [17] in defining $\mathbf{v} = \mathbf{u}/\lambda_1$ where $\nabla\cdot\mathbf{u} = 0$. This means that mixing conserves n/λ_1, not n: i.e.,

$$\partial_t(n/\lambda_1) + \mathbf{v}\cdot\nabla(n/\lambda_1) = 0. \tag{2}$$

Separating $n = \langle n\rangle + \tilde{n}$, we can write

$$\tilde{n} = -\tau_c\lambda_1\tilde{\mathbf{v}}\cdot\nabla(\langle n\rangle/\lambda_1),$$
$$\Gamma_n \approx \left\langle \lambda_1\tilde{v}_r\left(\frac{\tilde{n}}{\lambda_1}\right)\right\rangle \sim -\tau_c\left\langle |\tilde{v}_r|^2\lambda_1\frac{\partial}{\partial r}\left(\frac{\langle n\rangle}{\lambda_1}\right)\right\rangle, \tag{3}$$
$$\text{or}\quad V_r^{(n)} \approx -\frac{D_n}{\langle|\tilde{v}_r|^2\rangle}\left\langle |\tilde{v}_r|^2\lambda_1\frac{\partial}{\partial r}\left(\frac{1}{\lambda_1}\right)\right\rangle.$$

Here the flux has the form $\Gamma_n = -D_n\partial\langle n\rangle/\partial r + V_r^{(n)}\langle n\rangle$ and $D_n = \tau_c\langle|\tilde{v}_r|^2\rangle$, where τ_c is a turbulence decorrelation time. Note that $\langle n\rangle$ is, by definition, independent of the poloidal angle θ, $|\tilde{v}_r|^2$ and $\lambda_1 = \lambda_1(r,\theta)$ may not be.

Physically, the basis of the pinch is that the underlying physical processes (e.g., microturbulence) cause the particle number (the globally conserved quantity) to be homogenized among toroidal "shells," which means each shell ultimately contains an equal number of particles. However, since the volume of each shell is different, the density will be inversely proportional to the shell volume: $n \propto [V'(\psi)]^{-1}$. Here $V(\psi)$ is the total volume within the flux surface labeled by ψ. The volume of a shell of thickness Δr is approximately $V'(\psi)\Delta r$. For circular concentric flux surfaces $V'(r) \sim (2\pi)^2R_0 r$.

The form of the TEP theory that we use here is valid where the transport is turbulent drift-wave transport mixed via the self-consistent sheared flows, and the $E \times B$ shear effects, while dynamically important, are not strong as in a barrier. This is sufficient, for the effect is invoked to explain the pinch mechanism in the core region and not the generation in the pedestal. However, since the TEP is a geometry effect and is linked to diffusion, if one uses a more complex response (instead of just the τ_c) for the diffusive part (for instance, include $E \times B$ shear suppression), the same response applies to the pinch term as well.

The same is true for angular momentum density. It follows from the $Rv_\phi\hat{\phi}$ moment of the Vlasov equation that for $J_r = 0$ angular momentum density is exactly conserved (e.g., [1]):

$$\frac{\partial}{\partial t}(L_\phi) + \nabla\cdot(L_\phi\mathbf{v}) = 0, \tag{4}$$

which, in general, contains the fluctuations as well as the flux surface average. Letting $\mathbf{v} = \mathbf{u}/\lambda_1$ where $\mathbf{u}$ is divergence free (i.e., $\nabla\cdot\mathbf{u} = 0$) and $\lambda_1 = \lambda_1(\mathbf{r})$ is time independent, we arrive at the local angular momentum density evolution equation

$$\partial_t(L_\phi/\lambda_1) + \mathbf{v}\cdot\nabla(L_\phi/\lambda_1) = 0. \tag{5}$$

In other words, the locally conserved field is L_ϕ/λ_1 (e.g., for $\mathbf{v} = c\hat{\mathbf{z}}\times\nabla\Phi/B$ *à la* [18], $\lambda_1 = B$). Thus, for fluctuations, we can write

$$\tilde{L}_\phi = -\tau_c \lambda_1 \tilde{\mathbf{v}} \cdot \boldsymbol{\nabla}(\langle L_\phi \rangle / \lambda_1). \tag{6}$$

The mean field equation can be obtained by averaging (4):

$$\frac{\partial}{\partial t}\langle L_\phi \rangle + \frac{1}{r}\frac{\partial}{\partial r}[r\langle \tilde{L}_\phi \tilde{v}_r \rangle] = 0, \tag{7}$$

so that substituting (6) into (7) we get

$$\frac{\partial}{\partial t}\langle L_\phi \rangle - \frac{1}{r}\frac{\partial}{\partial r}\left(r\left[\left\langle \tau_c |\tilde{v}_r|^2 \lambda_1 \frac{\partial}{\partial r}\left(\frac{\langle L_\phi \rangle}{\lambda_1}\right)\right\rangle\right]\right) = 0. \tag{8}$$

Note that (8) has the form of divergence of a flux. This is because the total angular momentum is a globally conserved quantity (for $J_r = 0$), like particle number, and has the form

$$\frac{\partial}{\partial t}L_\phi - \frac{1}{r}\frac{\partial}{\partial r}(r\Pi_\phi) = 0,$$

where

$$\Pi_\phi \approx -\nu_T \frac{\partial}{\partial r}L_\phi - \frac{\nu_T}{\langle |\tilde{v}_r|^2 \rangle}\left\langle |\tilde{v}_r|^2 \lambda_1 \frac{\partial}{\partial r}\left(\frac{1}{\lambda_1}\right)\right\rangle L_\phi,$$

with turbulent viscosity (also called χ_ϕ) given by $\nu_T \sim \tau_c \langle |\tilde{v}_r|^2 \rangle$. In general, a more realistic kernel can be used, or collisional (molecular) viscosity in addition to turbulent viscosity may be considered. These are not essential for our current argument, and thus are excluded. The pinch velocity is

$$V_r^{(L)} = -\frac{\nu_T}{\langle |\tilde{v}_r|^2 \rangle}\left\langle |\tilde{v}_r|^2 \lambda_1 \frac{\partial}{\partial r}\left(\frac{1}{\lambda_1}\right)\right\rangle. \tag{9}$$

Thus, the physical basis of the "angular momentum pinch" is the same as the particle pinch. Instead of being diffused in the r coordinate, angular momentum is homogenized among infinitesimal shells by turbulent mixing. When each shell has equal "total" angular momentum, the process stops. Since the volumes of these shells are not equal, we have $\langle L_\phi \rangle \propto [V'(\psi)]^{-1}$. This is the origin of the TEP angular momentum pinch. Note that here we assumed the turbulence is electrostatic; hence, the field angular momentum remains constant overall. Note also that the angular momentum pinch as given in (9) relies on shaping as well as on whether the velocity fluctuations are ballooning or not. Equation (9) suggests that some ballooning is necessary. However, the mixing via self-consistent sheared flows, which enter the formulation by setting τ_c and by reducing the ballooning, are also dynamically important, as they enhance the mixing process responsible for the pinch.

In order to describe the evolution of "average linear momentum" in the ϕ direction, we use the relation between flux surface averaged angular and linear momenta: $\langle L_\phi \rangle = \langle I\omega_\phi \rangle \approx (\langle R^2 \rangle / \langle R \rangle)\langle P_\phi \rangle$, where I is the moment of inertia density. Noting that $\lambda_2 = \langle R \rangle / \langle R^2 \rangle$ is time independent, we can write

$$\frac{\partial}{\partial t}P_\phi - \lambda_2 \frac{1}{r}\frac{\partial}{\partial r}\left(r\left\langle \nu_T \lambda_1 \frac{\partial}{\partial r}\left(\frac{P_\phi}{\lambda_1 \lambda_2}\right)\right\rangle\right) = 0, \tag{10}$$

where $P_\phi \equiv m\langle n v_\phi \rangle$ and $\lambda_2 = mn\langle R \rangle V'(\psi)/I'_T(\psi) = \langle R \rangle / \langle R^2 \rangle$. Here $I_T(\psi)$ is the total moment of inertia of the plasma within a given flux surface ψ. Notice that, (10) is not exactly in the form of "divergence of a flux" due to an extra λ_2 outside the divergence in the second term. After moving λ_2 into the parenthesis by integrating by parts, part of the remaining term also takes the form of a divergence of a flux and can be combined with the first term. Consequently (10) can be expressed as

$$\frac{\partial}{\partial t}P_\phi - \frac{1}{r}\frac{\partial}{\partial r}\left(r\left\langle \nu_T \lambda_1 \lambda_2^2 \frac{\partial}{\partial r}\left(\frac{P_\phi}{\lambda_1 \lambda_2^2}\right)\right\rangle\right) = \gamma^{(P)} P_\phi, \tag{11a}$$

where

$$\gamma^{(P)} = \left\langle \frac{1}{\lambda_1 \lambda_2}\frac{1}{r}\frac{\partial}{\partial r}\left(r\nu_T \lambda_1 \frac{\partial \lambda_2}{\partial r}\right)\right\rangle \tag{11b}$$

and

$$\Gamma_\phi^{(p)} = -\nu_T \frac{\partial}{\partial r}P_\phi + V_r^{(p)} P_\phi. \tag{11c}$$

Here the pinch of linear momentum density is

$$V_r^{(P)} = -\frac{\nu_T}{\langle |\tilde{v}_r|^2 \rangle}\left\langle |\tilde{v}_r|^2 \lambda_1 \lambda_2^2 \frac{\partial}{\partial r}\left(\frac{1}{\lambda_1 \lambda_2^2}\right)\right\rangle. \tag{11d}$$

This form of the TEP pinch, which follows from the local conservation of $P_\phi / \lambda_1 \lambda_2^2$, can be directly deduced from (11a). Here $\gamma^{(P)}$ is the flux surface averaged manifestation of the fact that while angular momentum exchange between two fluid elements preserves total angular momentum, the total linear momentum may increase or decrease as a result of such exchange (for instance if angular momentum moves towards larger R, linear momentum has to decrease). Also noting that $\nu_T \sim \tau_c \langle |\tilde{v}_r|^2 \rangle$, for circular flux surfaces with positive or zero fluctuation intensity gradient, $\gamma^{(P)} < 0$.

As noted, it is the toroidal flow and not the momentum density that is measured in experiments. In order to write the evolution of toroidal flow, we need to disentangle the flow and the density. Assuming $P_\phi = \langle n \rangle \langle v_\phi \rangle$ in Eq. (11), we get

$$\frac{\partial}{\partial t}\langle v_\phi \rangle + \frac{\langle v_\phi \rangle}{\langle n \rangle}\frac{\partial}{\partial t}\langle n \rangle - \lambda_3 \frac{1}{r}\frac{\partial}{\partial r}\left(r\left[\nu_T \lambda_1 \frac{\partial}{\partial r}\left(\frac{\langle v_\phi \rangle}{\lambda_1 \lambda_3}\right)\right]\right) = 0,$$

where $\lambda_3 \equiv \lambda_2 / \langle n \rangle$. Using the equation of continuity and going through the same steps as for linear momentum, we obtain

$$\frac{\partial}{\partial t}\langle v_\phi \rangle + \frac{1}{r}\frac{\partial}{\partial r}(r\Gamma_\phi^{(v)}) = \gamma \langle v_\phi \rangle, \tag{12a}$$

where

$$\Gamma_\phi^{(v)} = -\nu_T \frac{\partial}{\partial r}\langle v_\phi\rangle + V_r^{(v)}\langle v_\phi\rangle \tag{12b}$$

$$V_r^{(v)} = -\frac{\nu_T}{\langle|\tilde{v}_r|^2\rangle}\left\langle|\tilde{v}_r|^2\lambda_1\lambda_3^2\frac{\partial}{\partial r}\left(\frac{1}{\lambda_1\lambda_3^2}\right)\right\rangle \tag{12c}$$

$$\gamma = \frac{1}{\lambda_1\lambda_3}\frac{1}{r}\frac{\partial}{\partial r}\left(r\nu_T\lambda_1\frac{\partial\lambda_3}{\partial r}\right) + \frac{1}{\langle n\rangle}\frac{1}{r}\frac{\partial}{\partial r}(r\Gamma_n)$$

with $\lambda_3 = \lambda_2/\langle n\rangle$ and $\Gamma_n = -D_n\lambda_1\frac{\partial}{\partial r}(\frac{\langle n\rangle}{\lambda_1})$.

The radial convection term is explicitly

$$V_r^{(v)} = V_r^{(P)} - 2\frac{\nu_T}{\langle n\rangle}\frac{\partial}{\partial r}\langle n\rangle, \tag{13}$$

where $V_r^{(P)}$ is given in (11d) and is always inward. The additional term is small if the density profile is flat (e.g., as in the core in H mode), but near the edge where the density gradient is large, the additional term (which is *outward*) may dominate. However, H mode sharp density gradient also implies a sharp temperature gradient, and there, the thermoelectric component [14] and the $E \times B$ shear also become important [10].

The primary results of this Letter [Eqs. (9), (11d), and (13)] are given in terms of λ_1 and λ_2. While λ_2 is given explicitly (i.e., as $\lambda_2 = \langle R\rangle/\langle R^2\rangle$), λ_1 is given only indirectly [i.e., $\nabla\cdot(\mathbf{v}_E\lambda_1) = 0$]. In a general sense, appearance of λ_1 in the angular momentum equation is a manifestation of the "frozen-in law" and the fact that density and angular momentum density are "linked." We can show the latter using the equation of continuity for a compressible plasma to write $\nabla\cdot\mathbf{v}_E \approx -(1/n)dn/dt$. This suggests that $d(L_\phi/n)/dt = 0$ (i.e., density and angular momentum density are linked). If the frozen-in law, i.e., $d(n/B)/dt = 0$ is applicable, this also implies $d(L_\phi/B)/dt = 0$. The connection between angular momentum density and density is general, but the frozen-in law relies on an equation of state, and thus on geometry, dimensionality, and/or dynamics. This links $\lambda_1 \propto n \propto V'(\psi)^{-1}$, and connects the coefficient to compressibility of the $\mathbf{E}\times\mathbf{B}$ flow due to geometry.

For simple slab geometry with an inhomogeneous magnetic field *à la* [18], $\mathbf{v}_E = \hat{\mathbf{z}}\times\nabla\Phi/B$, and so $\lambda_1 = B$ (leading to local conservation of L_ϕ/B). But for a low β torus, $\nabla\cdot(\mathbf{v}_E B^2) \rightarrow \mathbf{J}\cdot\nabla\Phi \ll B^2\nabla\Phi$, so we can use $\lambda_1 \approx B^2$, leading to local conservation of L_ϕ/B^2 instead. Note that characterizing ballooning structure of fluctuations via $F \cong \langle|\tilde{v}_r|^2(\cos\theta + \hat{s}\sin\theta)\rangle/\langle|\tilde{v}_r|^2\rangle$, one can recover $V_r^{(L)} = -2\chi_\phi F/R_0$, the same expression for the TEP part of the pinch as in [14]. The same assumptions (with concentric circular flux surfaces) lead to $V_r^{(P)} = -2\chi_\phi(F + \epsilon)/R$, where $\epsilon = r/R_0$. This means that some pinch effect that comes from the radial profile of moment of inertia in a torus persists even if $F = 0$ (flute limit), for linear momentum in the toroidal direction. Furthermore, for toroidal flow, one may use Eq. (13) and obtain $V_r^{(v)} = 2\chi_\phi[-(F + \epsilon)/R + L_n^{-1}]$. Note that a better understanding of the effects of density gradient on flow pinch requires a detailed study of the particle transport and, in particular, a realistic electron response.

In this Letter, we present a simple interpretation of the TEP pinch of angular momentum. The resulting angular momentum pinch agrees with the TEP part of the total pinch that was derived previously using gyrokinetic formulation. We also show that for average linear momentum, there is an additional term related to the radial profile of moment of inertia density in a torus.

We thank J. Rice, A. Ince-Cushman, M. Yoshida, Y. Kamada, V. Naulin, X. Garbet, W. M. Solomon, J. S. deGrassie, A. Bortolon, and B. P. Duval for stimulating discussions. This work was supported by U.S. DOE Grant No. FG02-04ER-54738 (UCSD) and U.S. DOE Contract No. DE-AC02-76-CH0-3073 (PPPL).

*ogurcan@ucsd.edu

[1] F. L. Hinton and S. K. Wong, Phys. Fluids **28**, 3082 (1985).
[2] N. Mattor and P. H. Diamond, Phys. Fluids **31**, 1180 (1988).
[3] S. D. Scott *et al.*, Phys. Rev. Lett. **64**, 531 (1990).
[4] J. E. Rice *et al.*, Nucl. Fusion **44**, 379 (2004).
[5] J. E. Rice *et al.*, Fusion Sci. Technol. **51**, 288 (2007).
[6] J. S. deGrassie *et al.*, Phys. Plasmas **14**, 056115 (2007).
[7] W. M. Solomon *et al.*, Plasma Phys. Controlled Fusion **49**, B313 (2007).
[8] M. Yoshida *et al.*, Plasma Phys. Controlled Fusion **48**, 1673 (2006).
[9] A. Bortolon, B. P. Duval, A. Pochelon, and A. Scarabosio, Phys. Rev. Lett. **97**, 235003 (2006).
[10] Ö. D. Gürcan, P. H. Diamond, and T. S. Hahm, Phys. Plasmas **14**, 055902 (2007).
[11] P. H. Diamond, C. J. McDevitt, Ö. D. Gürcan, T. S. Hahm, and V. Naulin, Phys. Plasmas **15**, 012303 (2008).
[12] T. S. Hahm, Phys. Fluids **31**, 2670 (1988).
[13] T. S. Hahm, Phys. Plasmas **3**, 4658 (1996).
[14] T. S. Hahm, P. H. Diamond, O. D. Gurcan, and G. Rewoldt, Phys. Plasmas **14**, 072302 (2007).
[15] A. G. Peeters, C. Angioni, and D. Strintzi, Phys. Rev. Lett. **98**, 265003 (2007).
[16] M. B. Isichenko, A. V. Gruzinov, and P. H. Diamond, Phys. Rev. Lett. **74**, 4436 (1995).
[17] V. Naulin, J. Nycander, and J. J. Rasmussen, Phys. Rev. Lett. **81**, 4148 (1998).
[18] X. Garbet *et al.*, Phys. Plasmas **12**, 082511 (2005).
[19] P. B. Rhines and W. R. Young, J. Fluid Mech. **122**, 347 (1982).
[20] P. H. Diamond, S. I. Itoh, K. Itoh, and T. S. Hahm, Plasma Phys. Controlled Fusion **47**, R35 (2005).
[21] H. Biglari, P. H. Diamond, and P. W. Terry, Phys. Fluids B **2**, 1 (1990).